Vectors, lookups, and interpolation

A vector is a special case of a matrix. There is one column in a vector.

Creating a vector: Use the insert matrix command $V1 := \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $V2 := \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

Dot product: $V1 \cdot V2 = 22$

Note that the dot product does not work with vectors defined as rows:

$V3 := (1 \ \ 2)$ $V4 := (2 \ \ 4)$ $V3 \cdot V4 = \blacksquare$ Does not work.

Cross product: $Z := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $Y := \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $Z \times Y = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$ $Y \times Z = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

The cross product does not work unless the vectors have three elements:

$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \blacksquare$ Does not work.

Vector with defined elements: $k := 0..7$ $XX_k := \text{ceil}\left(\frac{k}{2}\right)$ $YY_k := 2^{\text{ceil}\left(\frac{k}{2}\right)}$

$XX^T = (0 \ \ 1 \ \ 1 \ \ 2 \ \ 2 \ \ 3 \ \ 3 \ \ 4)$

$YY^T = (1 \ \ 2 \ \ 2 \ \ 4 \ \ 4 \ \ 8 \ \ 8 \ \ 16)$

Transpose is displayed here because it occupies less spreadsheet space.
(The *ceil* function rounds a number up).

Match: The function **match** returns the index corresponding to a value

$\text{match}(8, YY) = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$ because the 5th and 6th elements of array YY are both 8.

An exact match is required. **Match** will not find the closest value.

Lookup: which XX values above have the same position as the YY values 8?

$\text{lookup}(8, YY, XX) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ The parentheses around the answer indicate a matrix or vector has been returned as the answer. **lookup** will list several values for XX if 8 appears more than once in YY. To use the first value returned, select the zeroth component of the vector.

$\text{lookup}(1, YY, XX) = (0)$ If there is only one value, it is returned in a matrix of one element.

Lookup converted from matrix to scalar: $\text{lookup}(1, YY, XX)_0 = 0$

Linear interpolation To demonstrate interpolation, we need to set up two data arrays

$k := 0..7$ $XX_k := 2\cdot(k+1)$ $YY_k := [2\cdot(k+1)]^2$

$XX^T = (2\ \ 4\ \ 6\ \ 8\ \ 10\ \ 12\ \ 14\ \ 16)$ Even numbers.

$YY^T = (4\ \ 16\ \ 36\ \ 64\ \ 100\ \ 144\ \ 196\ \ 256)$ Squares of the even numbers.

$\mathrm{linterp}(XX, YY, 5) = 26$ Square of the odd number 5, by linear interpolation.
It is inaccurate because the square is not a linear function.

Spline interpolation: Between each adjacent pair of points, a cubic curve is fitted, using nearby points to make the curve smooth.

For spline interpolations, we need to find the vector *vs* of coefficients for the spline curves:

$vs := \mathrm{cspline}(XX, YY)$ Finds the coefficients *vs* needed by *interp*.

$\mathrm{interp}(vs, XX, YY, 5) = 25$ Interpolation by cubic splines. In this example the interpolated answer is precise because a third order polynomial can be fit to a parabola exactly.

Extrapolation using splines: $\mathrm{interp}(vs, XX, YY, 17) = 289$ Here we have extrapolated to find the square of 17.

If a third order polynomial is a poor fit to the data, then the extrapolation will not be accurate.

To find a maximum in a vector or matrix: $\max(XX) = 16$ $\max(YY) = 256$

To find a minimum: $\min(XX) = 2$ $\min(YY) = 4$

Reversal of the order of terms:

$$XX = \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \\ 16 \end{pmatrix} \qquad \mathrm{reverse}(XX) = \begin{pmatrix} 16 \\ 14 \\ 12 \\ 10 \\ 8 \\ 6 \\ 4 \\ 2 \end{pmatrix}$$