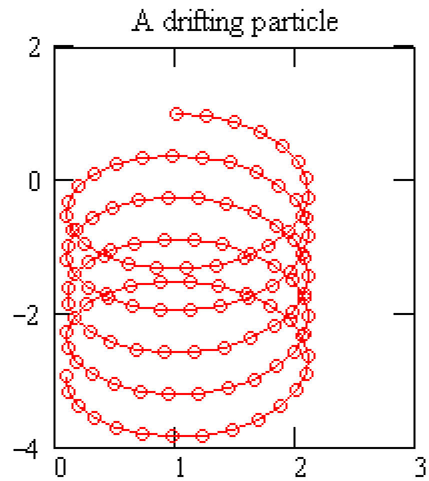
E x B drift in uniform fields

This exercise explores the motion of a charged particle in a uniform magnetic field when there is also a uniform electric field. The particle will drift perpendicular to E and B.



ORIGIN := 1

ORIGIN = 1 specifies that the vector components are E_1, E_2 and E_3 and not E_0, E_1, E_2.
We will define a 6-vector Z that will have subscripts 1,2 and 3 for x,y, and z and 4,5,6 for Vx, Vy, Vz.

$$Z := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{matrix} X \\ Y \\ Z \\ Vx \\ Vy \\ Vz \end{matrix}$$

Our 3 vector components of B are

$$B := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{matrix} B_x \\ B_y \\ B_z \end{matrix}$$

Our three vector components of E are

$$E := \begin{pmatrix} 0.1 \\ 0 \\ 0 \end{pmatrix}$$

The E_x will give us a drift in the y direction.

The equations of motion

Pick starting X and V vectors and a starting time.

$$X := \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad V := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad t := 0$$

We will stack these values and put them into our 6-vector Z: $Z := \text{stack}(X, V)$

The next two definitions let us recover X and V from the stack Z:

$X(Z) := \text{submatrix}(Z, 1, 3, 1, 1)$

$V(Z) := \text{submatrix}(Z, 4, 6, 1, 1)$

We will avoid scientific notation by defining:

$q := 1$ $\qquad$ $m := 1$

At right, DZ is the time derivative of the 6-vector Z. The first three terms are the derivatives of x, y, and z and the second three terms are the time derivatives of Vx, Vy, and Vz. These definitions are the **Lorentz equations of motion**.

$$DZ(t, Z) := \begin{bmatrix} Z_4 \\ Z_5 \\ Z_6 \\ \frac{q}{m} \cdot \left[E + (V(Z) \times B)\right]_1 \\ \frac{q}{m} \cdot \left[E + (V(Z) \times B)\right]_2 \\ \frac{q}{m} \cdot \left[E + (V(Z) \times B)\right]_3 \end{bmatrix} \quad \begin{matrix} dx/dt \\ dy/dt \\ dz/dt \\ dVx/dt \\ dVy/dt \\ dVz/dt \end{matrix}$$

Now find and plot the particle trajectory

The gyro frequency is $\Omega := \frac{q}{m} \cdot B_3$ $\Omega = 1$

Our time interval should be divided more finely than the gyration period.

Let t be the total time interval $t := 30$

The number of iterations we will use is about 4 per gyro period or 8 π per orbit.

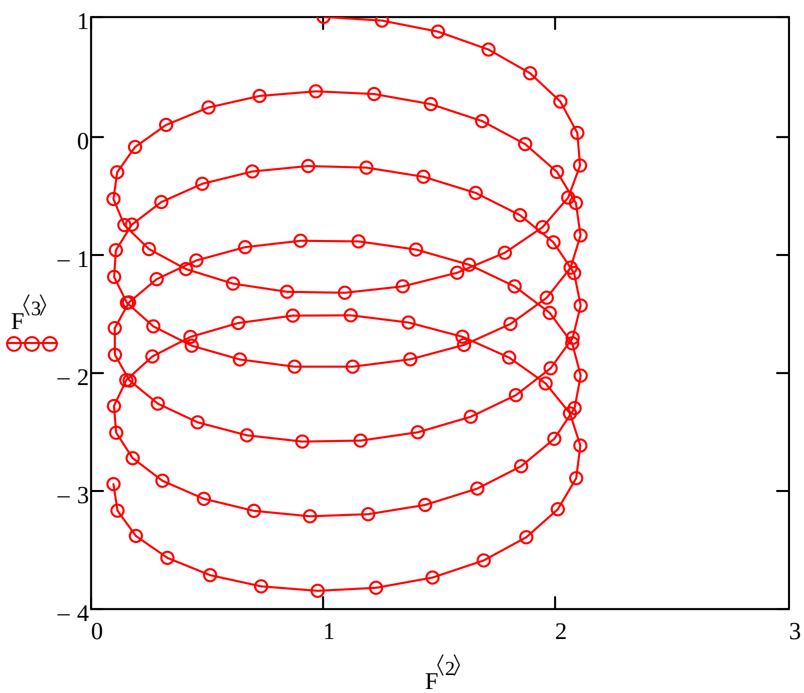
$npoints := ceil(4 \cdot |\Omega| \cdot t)$ ceil is a rounding function to make integers

$npoints = 120$ This is calculated, not guessed. It is always a good practise to base npoints on something other than a wild guess. We have made time interval dt of 1/4 of the characteristic time of the problem, in this case 1/4Ω. The number of these intervals in the time t is equal to 4 Ω t.

Now integrate:

$F := rkfixed(Z, 0, t, npoints, DZ)$

This view looks parallel to B. It is the motion in the x,y plane:



$F^{<2>}$ is x and $F^{<3>}$ is y.

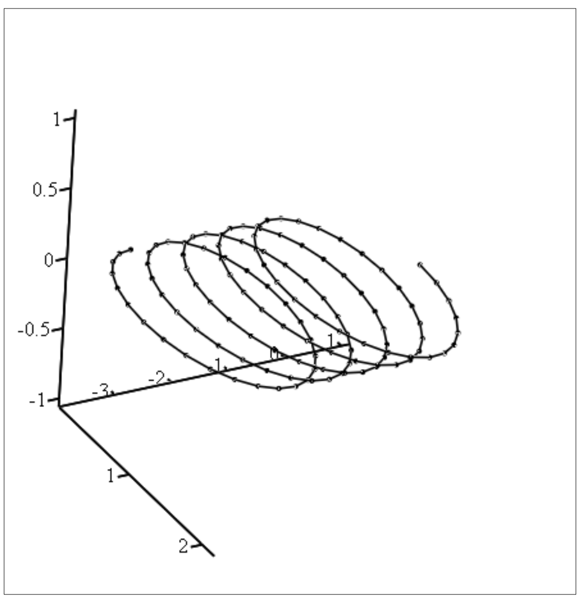
This is the trajectory in our 6-dimensional space:

F =

	t	x	y	z	Vx	Vy	Vz
	1	2	3	4	5	6	7
1	0	1	1	0	1	0	0
2	0.25	1.251	0.969	0	0.994	-0.251	0
3	0.5	1.492	0.876	0	0.926	-0.492	0
4	0.75	1.708	0.725	0	0.8	-0.708	0
5	1	1.887	0.524	0	0.624	-0.887	0
6	1.25	2.017	0.285	0	0.41	-1.017	0
7	1.5	2.09	0.021	0	0.171	-1.09	0
8	1.75	2.102	-0.255	0	-0.08	-1.102	...

Click on the table to show the scroll bar.

An interactive 3-d view, just grab a corner and pull:



$\left(F^{\langle 2\rangle}, F^{\langle 3\rangle}, F^{\langle 4\rangle}\right)$

The motion above is the typical E x B drift.

Try it: What happens when you set the initial x velocity to zero? To 6?

Try it: What happens when E_z is changed from 0 to 0.1?