A gyrating particle: a video

This exercise demonstrates the use of Runge Kutta to solve the Lorentz equations of motion.
We will let q = m = 1 so that we don't need scientific notation to display the times and distances:

$q := 1$ $m := 1$ Charge and mass of the particle.

We will let the B field = 1 and assume this is in the z direction. $B := 1$

The velocity will be a 2-vector:

$V := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{matrix} V_x \\ V_y \end{matrix}$

To create this vector, on the right side of := select a matrix from the matrix menu and make it one column wide and two rows high. A vector is a matrix with one column.

We can check on what we did by asking what are the zeroth and first components of the vector. Mathcad numbers things starting with zero, usually. We can force Mathcad to start numbering at 1 by using the ORIGIN:=1 statement.
But this is not always a good idea. Some features of Mathcad don't work as expected when the ORIGIN is moved.

$V_0 = 0$

$V_1 = 1$

Here is the Lorentz equation of motion:

The 2-vector DV holds the derivatives of V. Mathcad requires that DV be written as a function of t as well as V.

$$DV(t,V) := \begin{bmatrix} \frac{q}{m} \cdot V_1 \cdot B \\ -\left(\frac{q}{m} \cdot V_0 \cdot B\right) \end{bmatrix}$$

same as:

$$\frac{q}{m} \cdot v_y \cdot B_z$$

$$\frac{-q}{m} \cdot v_x \cdot B_z$$

We use the Runge-Kutta method to find the solution by iteration. We will start at t = 0 and go to t = 12. The gyration or cyclotron frequency is 1 so at t = 6.28 we have done one revolution. Accuracy will be poor if the particle goes too far between iterations. The natural units for distance around an orbit are radians. So we want to make our time steps smaller than a radian of gyration. We will choose 1/4 gyroperiod as the time step and then we will need about 50 points.

$t := 12$ $\Omega := \frac{q}{m} \cdot B$ $\Omega = 1$ $npoints := ceil(4\,|\Omega| \cdot t)$ ceil rounds up a number

$npoints = 48$

It is always a good idea to calculate npoints rather than guess. This will cause npoints to remain correct if you change q, B or m.

Now apply Runge-Kutta and put the answers in a matrix F.

$F := rkfixed(V, 0, t, npoints, DV)$

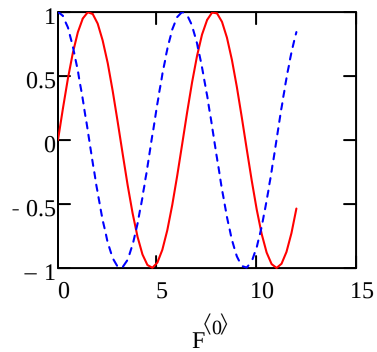
The matrix F has t, V_x, and V_y in columns 0, 1, and 2.

t V_x V_y

F =

	0	1	2
0	0	0	1
1	0.25	0.247	0.969
2	0.5	0.479	0.878
3	0.75	0.682	0.732
4	1	0.841	0.54
5	1.25	0.949	0.315
6	1.5	0.997	0.071
7	1.75	0.984	-0.178
8	2	0.909	-0.416
9	2.25	0.778	-0.628
10	2.5	0.599	-0.801
11	2.75	0.382	-0.924
12	3	0.141	-0.99
13	3.25	-0.108	-0.994
14	3.5	-0.351	-0.936
15	3.75	-0.571	...

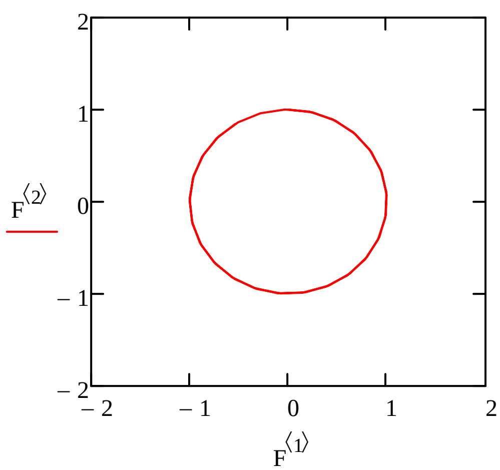
an see in the matrix and in the graph
that the velocity components oscillate
en +1 and -1 and are out of phase. One
as the cosine and the other as the sine.



The zeroth columns of F, written $F^{<0>}$, is time. $F^{<1>}$ is V_x and $F^{<2>}$ is V_y.

The velocities can be plotted with V_y on the vertical axis and V_x on the horizontal axis. This is done by selecting the appropriate column in the F matrix.

This is a plot of $V_y(t)$ as a function of $V_x(t)$.

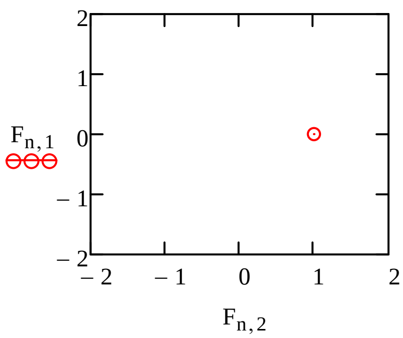


A movie of velocity as a function of time:

To make a movie of the velocity (not the position) as a function of time, first make a plot of the first value taken by V_x and V_y.

$FRAME = 0$ initialize frame

$n := FRAME$



Set n equal to the FRAME number. FRAME is the variable Mathcad uses to number the frames of the movie. As n = FRAME advances, successive values of F are plotted.

<u>How to create a movie:</u>

1. Put the data in a vector or matrix.
2. Make a plot of the first point. The point should be labeled by a subscript, for example "n".
3. The subscript should be set equal to FRAME at a place just above the graph. FRAME will have the value zero before the movie is made.
4. Choose View|Animate and a dialog box appears.
5. Fill in the values to be used for FRAME, for example 0 to 50. A longer movie than 50 FRAMES will take a longer time to generate.
6. Use the mouse to put a box around the graph. What appears in the box will appear in the animation. Select Animate.
7. The animation will appear in the Animate dialog box and again in a Playback box.
8. You can test the animation by clicking start in the Playback box.
9. In the Animation box, choose Save As, give the animation a name, and save it.
10. Insert the animation in the Mathcad sheet by choosing Insert|Object|Video Clip (near the bottom of the list, you'll have to scroll down). Choose Create From File. And Browse until you find your animation. Select it and it will appear.