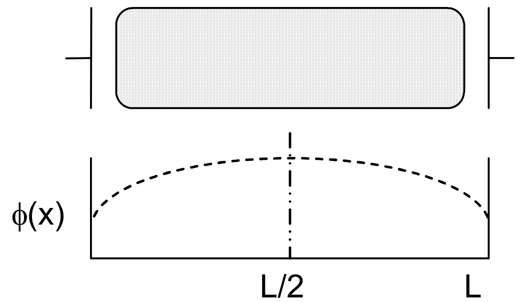
Source sheath and collector sheath in the Q machine

The Q machine is a device in which metal vapor is sprayed onto a hot surface (the hot plate). The metal vapor is ionized by the hot plate and the resulting ions leave the surface. The hot plate also emits electrons thermionically so that both electrons and ions are produced, but not necessarily at the same rate. The surface is usually a metal of high melting point (such as tungsten or tantalum) and the metal vapor is usually an alkali metal (such as cesium or potassium) with a low ionization potential. In the double-ended Q machine there is a plasma source at each end and an axial magnetic field to prevent loss of plasma to the cylindrical walls. A sheath forms at the hot plate so that electron and ion densities in the plasma are equal.



The double-ended Q machine

In **part I,** we will solve for the potential profile of the double-ended Q machine using Poisson's equation. We will apply periodic boundary conditions at the right boundary at distance L/2. This means that if the two sources (facing one another) are separated by length L, the region modeled is half the device. The source will be at the left boundary and the symmetry condition will be applied at the right boundary.

In **part II**, we will analyze a single-ended Q machine with a source at one end and a collector at the other end. If the collector is not electrically connected to the source, it will float to the potential that equalizes collection of electrons and ions. In this case there is a sheath at the collector as well as a sheath at the source. In part II, we will find both the source sheath and the collector sheath.

These problems are more challenging than the problem of the sheath above an electron emitter because there are both electrons and ions and there are both maxima and minima in the potential.

Part I. Double-ended Q machine with source sheath

Method for solving Poisson's equation

Poisson's equation will be solved by successive approximations using the finite-differencing method introduced in the exercise "Poisson's Equation and Debye Shielding." Familiarity with that exercise is assumed. Also we use the dimensionless units from that exercise in which the Debye length for electrons is the length scale. The iteration method is

$$\Phi_k \leftarrow \frac{1}{2}\left[\Phi_{k+1} + \Phi_{k-1}\right] + \frac{1}{2}(\Delta x)^2\left[n_i(x_k) - n_e(x_k)\right]$$

The left arrow indicates that the present value of $\Phi(x_k)$ is to be replaced by the value on the right hand side. This procedure is repeated until the solution for Φ has "relaxed" to a final value. The densities of electrons and ions are found as a function of the potential $\Phi(x_k)$ also written Φ_k. An initial guess is used for Φ_k to start the iteration procedure.

Periodic boundary conditions

In our model of the double-ended Q machine, beyond the right boundary there is a mirror image of the plasma and the source. The last grid point will be x_{kmax}. The algorithm above for solving Poisson's equation requires Φ_{k+1} to calculate the new Φ_k. Thus at x_{kmax} we will need to know $\Phi(x)$ at x_{kmax+1} which is beyond the boundary. By symmetry, this is the same value of potential that is at x_{kmax-1}. Hence, at the right boundary the numerical algorithm above changes to:

$$\Phi_{\text{kmax}} \leftarrow \Phi_{\text{kmax}-1} + \frac{1}{2}(\Delta x)^2 \left[n_i(x_k) - n_e(x_k) \right]$$

The physical origin of the sheath potential

The electron and ion distributions directly from the hot plate are assumed to be a single-sided Maxwellians. The temperatures of the electrons and ions are both equal to the temperature of the hot plate. The electrons and ions may be emitted at different rates thus the sheath adjacent to the plate may be electron-rich or ion-rich. We will investigate ion-rich conditions. A sheath with a positive potential forms that returns a fraction of the ions to the surface so that the electron and ion densities are the same in the bulk of the plasma.

Dimensionless units

$$\tilde{\Phi} = q\Phi / T$$

$$\tilde{x} = \frac{x}{\sqrt{\varepsilon_0 T / n_0 q^2}} = \frac{x}{\lambda_{Debye}}$$

$$\tilde{n} = n / n_0$$

$$\tilde{v} = \frac{v}{\sqrt{2T_e / m_e}}$$

These are the dimensionless units for potential, distance, density, and velocity.

$$\frac{d^2}{d\tilde{x}^2}\tilde{\Phi}(\tilde{x}) = \tilde{n}_i(\tilde{x}) - \tilde{n}_e(\tilde{x})$$

This is Poisson's equation in dimensionless units.
Below, we use these dimensionless units without the tildes.

The model for the electron density and current

The positive plasma potential of an ion-rich sheath accelerates emitted electrons away from the source. The electron distribution above the hot plate at the left boundary consists of emitted electrons moving to the right and of electrons moving to the left from the hot plate at the right boundary. If the local potential is φ relative to the grounded hot plate, and φ > 0, then there are no electrons with velocity less than

$$v_{min}(\phi) := \sqrt{\phi}$$

for $\phi > 0$, where ϕ is the local potential.

A one-dimensional Maxwellian for the electrons with density n_{e0} = 1 is $n_{e0} := 1$

$$f_e(v,\phi) := \frac{n_{e0}}{\sqrt{\pi}} \cdot \exp\left(-v^2 + \phi\right)$$

For ϕ > 0, the electron density at potential ϕ is: $n_e(\phi) := 2 \cdot \int_{\sqrt{\phi}}^{\infty} f_e(v,\phi)\, dv$

where the factor of two is in front because both left-going and right-going electrons are assumed for a double-ended machine.

The electron density at zero potential is normalized to unity: $n_e(0) = 1$

At zero potential, the current of electrons moving to the right is found from the first moment of the electron distribution:

$$J_e := \int_0^{\infty} f_e(v,0) \cdot v\, dv \qquad J_e = 0.282$$

This value agrees with the analytic result for a Maxwellian distribution: $\frac{1}{2} \cdot \sqrt{\frac{1}{\pi}} = 0.282$

The dimensionless flux agrees with equations 7-11 and 7-15 in Chen's textbook.

The model for the ion density and current

Ion-rich conditions will be created by setting the ion density at zero potential equal to ten times the electron density.

$n_{i0} := 10 \cdot n_{e0}$ This definition creates ion-rich conditions.

The ions and electrons are created at the temperature of the hot plate, hence $T_i = T_e$.

The ion distribution function is then:
(in dimensional units)

$$f_i(v,\phi) := n_{i0} \cdot \sqrt{\frac{m_i}{2 \cdot \pi \cdot T_i}} \cdot \exp\left(-\frac{m_i \cdot v^2}{2 \cdot T_i} - \frac{q \cdot \phi}{T_i}\right)$$

Let $\alpha := \frac{m_i}{m_e}$ $\alpha := 1836$ For hydrogen.

The dimensionless ion distribution is: $f_i(v,\phi) := n_{i0} \cdot \sqrt{\frac{\alpha}{\pi}} \cdot \exp\left(-\alpha \cdot v^2 - \phi\right)$

An ion with energy ϕ will have velocity $(\phi/\alpha)^{1/2}$.

In the plasma at positive potential ϕ, ions with insufficient energy are returned to the source. For ion energies that are reflected, there are emitted ions moving to the right and reflected ions moving to the left. At higher energies ions are not reflected, however, there are ions moving to the left that have their origin at the source on the right side and have passed over the potential hill at x = L/2. Thus there is no range of energy that is missing for ions moving in either direction. After the integral over velocities is performed, the ion density is:

$$n_i(\phi) := 2\cdot\int_0^{\infty} f_i(v,\phi)\,dv \qquad n_i(0) = 10$$

This is the ion density that we specified above to create ion-rich conditions.

At the hot plate surface, the potential is zero and the ion current is

$$J_i := \int_0^{\infty} f_i(v,0)\cdot v\,dv \qquad J_i = 0.066$$

We expect J_i to be larger than J_e by the factor n_i/n_e (the density ratio) and smaller by the factor $1/\alpha^{1/2}$ because of the thermal velocity ratio.

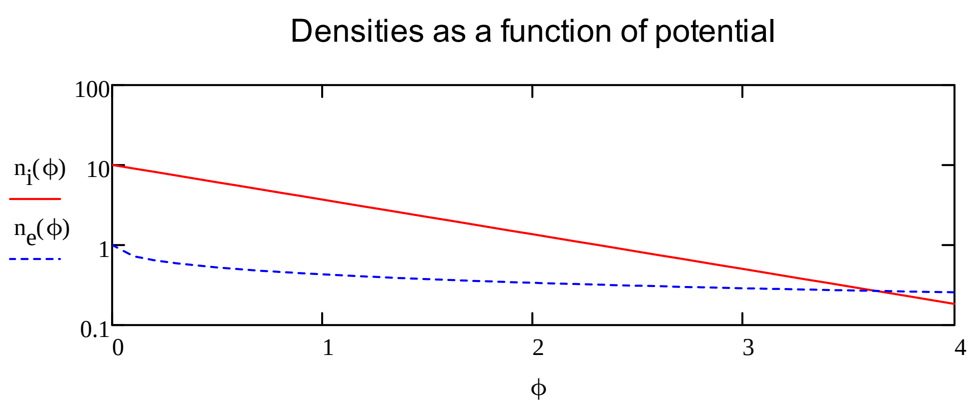
$$J_e\cdot\frac{n_{i0}}{n_{e0}}\cdot\frac{1}{\sqrt{\alpha}} = 0.066$$

This shows that J_i what is expected.

The plasma potential

The plasma potential will have the value that results in equal densities of electrons and ions. We find the potential by plotting the electron and ion densities as a function of ϕ and noting where the curves cross:

$\phi := 0, 0.1 .. 4$ Range of potentials for plotting.



The curves appear to cross at about 3.6.

We will use the root function to find the curve crossing (the plasma potential) more accurately:

$\phi := 3$ Initial guess for ϕ.

$\phi_{plasma} := \text{root}\left[\left(n_e(\phi) - n_i(\phi)\right), \phi\right]$

$\phi_{plasma} = 3.627$

This is the plasma potential that equalizes the densities for our ion-rich conditions.

Define the x grid

Δx is the spacing in the x direction. For the plasma potential defined above, the density is less than unity and the Debye length is greater than unity. We will make an initial guess that $\Delta x = 1$ is sufficient to resolve details and provide stability.

$xmax := 40$ The width of the domain

$\Delta x := 1$ The specified grid spacing.

$kmax := \dfrac{xmax}{\Delta x}$

$k := 0, 1 .. kmax$

$x_k := k \cdot \Delta x$ There will be kmax+1 grid points.

These grid points define a domain that is approximtaely 40 Debye lengths long. We are unsure of the Debye length at this point because we do not yet know the central plasma density.

Solution to Poisson's equation by successive approximations

The successive Φ values are saved in a matrix Phi. Each row in the matrix will be an iteration. Saving the intermediate interations will allow us to examine convergence.

We will make an initial guess that we will need 100 iterations and will check later to be sure that this number is sufficient.

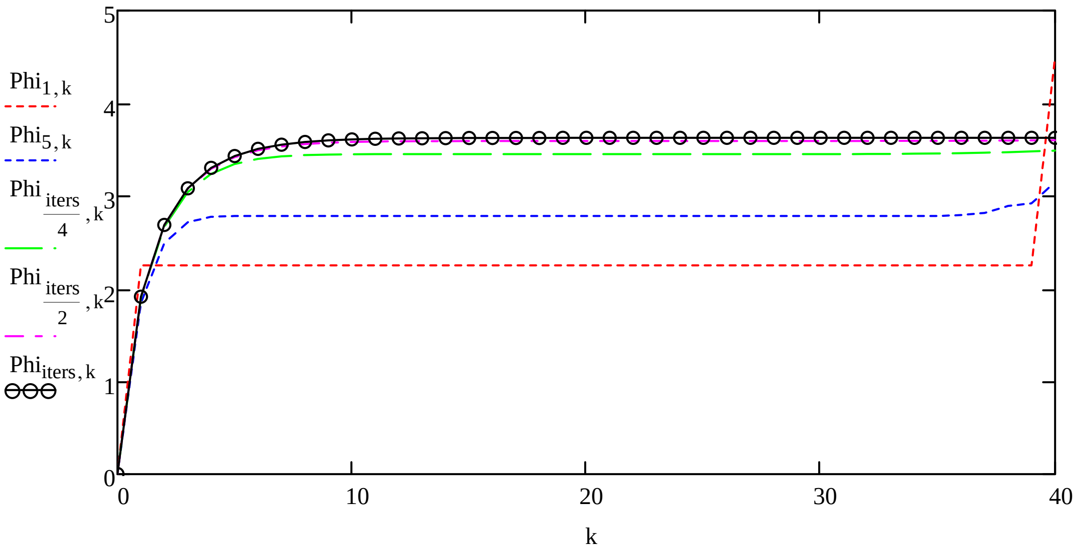
$iters := 100$ The number of iterations. The plot below requires that iters be divisible by 4.

The program loop below begins by initializing all rows of Phi to zero. The zeroth row of Phi is used to start the iteration procedure.

```
Phi := | Phi[iters, kmax] ← 0
       | for i ∈ 1 .. iters
       |   | for k ∈ 1 .. kmax − 1
       |   |   | C ← 0.5·(Phi[i−1, k+1] + Phi[i−1, k−1]) + 0.5·Δx^2·(−n_e(Phi[i−1, k]) + n_i(Phi[i−1, k]))
       |   |   | Phi[i, k] ← 0.5·(C + Phi[i−1, k])
       |   | Phi[i, kmax] ← Phi[i−1, kmax−1] + 0.5·Δx^2·(−n_e(Phi[i−1, kmax]) + n_i(Phi[i−1, kmax]))
       |   | Phi[i, 0] ← Phi[i−1, 0]
       | Phi
```

The last line in the "for i" loop preserves the boundary condition at x = 0.
The line before that applies the periodic boundary condition at kmax.
The iterative technique is stable if the new answer is averaged with the old answer before it is used. The averaging is at the end of the "for k" loop. The variable C holds the value before it is averaged. This averaging is discussed in the exercise "Poisson's Equation and Debye Shielding."

A plot of Phi for several iterations



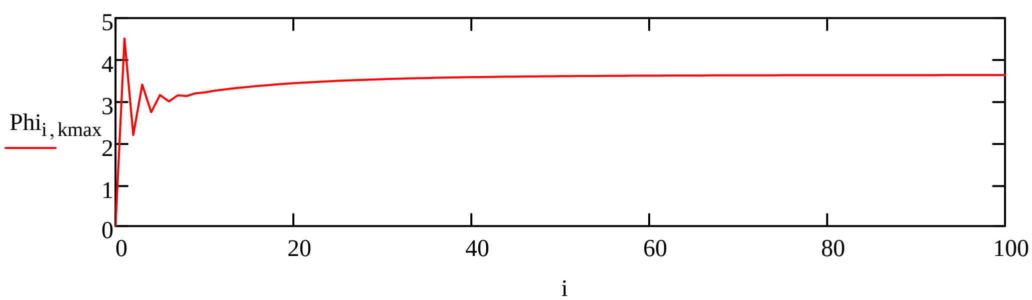
Ion-rich conditions at the source have resulted in a positive plasma potential for equal densities of electrons and ions.

Test for convergence

Did we use a large enough value for iters to obtain convergence to the correct answer?
We can answer this question by watching the evolution of Φ at the right boundary.

$i := 0 .. iters$ Index for plotting successive value of Φ_{kmax}.

This plot shows convergence to a final value for Φ_{kmax} indicating that our value for iters was sufficient:



Is the final value for the plasma potential the correct value?
We compare the final value from the iterations to the value from the root finder:

The final value for Φ_{kmax} is: $Phi_{iters,kmax} = 3.626$

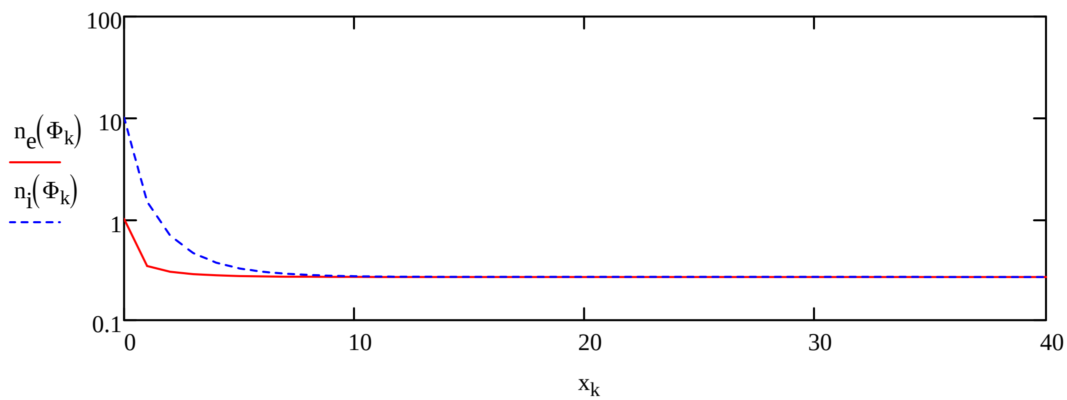
This value agrees with the value found by the root finder: $\phi_{plasma} = 3.627$

Plot of the number densities

The densities of electrons and ions are a function of the potential. In order to plot the densities, we first need to put the final answer for Φ into a vector. We will transpose the answer matrix Phi and use the last iteration for Φ.

$\Phi := \left(Phi^T\right)^{\langle iters \rangle}$ Note that the value at the right boundary is: $\Phi_{kmax} = 3.626$

Log plot of electron and ion densities for ion-rich conditions



At the right boundary, the electron and ion densities are nearly equal indicating a quasineutral plasma:

$n_e(\Phi_{kmax}) = 0.266$ $n_i(\Phi_{kmax}) = 0.266$ $n_i(\Phi_{kmax}) - n_e(\Phi_{kmax}) = 2.82 \times 10^{-4}$

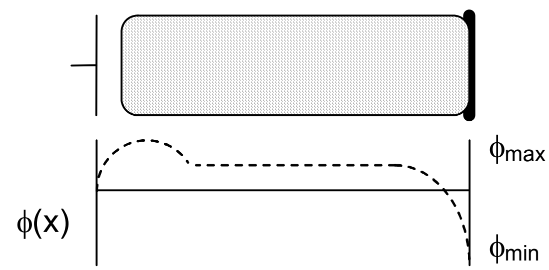
The revised Debye length

The Debye length that we used to make the equations dimensionless was based on the electron density which was specified as n_{e0} = 1. However, the final plasma has density 0.266. Hence we must be careful in using our results. The horizontal scale of our graphs is not the Debye length of the quasineutral plasma column.

It is tempting to change the ion density n_{i0} to a value less than 1 to investigate **electron-rich conditions**, however, this change will give wrong answers because the expressions for the electron and ion densities only apply for positive potentials.

Part II. Single-ended Q machine with source and collector sheaths

The single-ended Q machine has a collector at the right boundary that is often floating electrically. A floating collector charges until the currents of electron and ions are equal. The collector potential $q\phi_{min}$ becomes the most negative potential in the device. This potential repels electrons with energy less than $-\phi_{min}$ (assuming $\phi_{min} < 0$). Thus the electron distribution has electrons going in both directions only for energies below $-q\phi_{min}$. We begin by modifying our expressions for the electron and ion densities and currents to acount for the presence of the collector and the nonmonotonic potential profile.



Single-ended Q machine with floating collector

We again assume ion-rich conditions which create a potential maximum ϕ_{max} near the emitter.

The ion density and current

First consider the region to the left of the potential maximum ϕ_{max} which wll be called **region 1**. At the source, the distribution of ions has reflected ions at energies below the energy ϕ_{max}. At other locations, reflection ions have energies up to ϕ_{max}-ϕ. This potential will be found in the program loop using the **max** function. At higher energies, the ions pass over the potential hill and are collected at the right boundary. The ion density in region 1 must be calculated by assuming a two-sided distribution at energies below ϕ_{max} and a one-sided distribution at higher energies. The expression for the ion density in region 1 is then

$$n1_i(\phi, \phi_{max}) := \left(2 \cdot \int_0^{\sqrt{\frac{\phi_{max}-\phi}{\alpha}}} f_i(v,\phi)\, dv + \int_{\sqrt{\frac{\phi_{max}-\phi}{\alpha}}}^{\infty} f_i(v,\phi)\, dv \right)$$

Note that the first integral has a factor of two in front because these lower energy ions are reflected and count twice toward the density.

This expression should give a density near n_{i0} if ϕ = 0 and ϕ_{max} = 5: $n1_i(0,5) = 9.992$

The result above is near to the value that we expect: $n_{i0} = 10$

If the potential is more positive than zero, the density becomes smaller because of the low-energy ions that did not make it to the point of observation: $n1_i(2,5) = 1.344$

Let the x value at ϕ_{max} be x_{max}. Then $x > x_{max}$, will be called **region 2**. There are no left-going ions in region 2. Ions in this region are accelerated toward the collector. In region 2, the ions that have come over the potential hill and that have been accelerated have a minimum velocity of

$$v_{min}(\phi,\phi_{max}) := \sqrt{\frac{\phi_{max} - \phi}{\alpha}}$$

The velocity integrals in region 2 must be started at this velocity.

The expression for the ion density in region 2 is:

$$n2_i(\phi,\phi_{max}) := \int_{\sqrt{\frac{\phi_{max}-\phi}{\alpha}}}^{\infty} f_i(v,\phi)\, dv$$

We can test these expressions as follows. Regions 1 and 2 come together at the potential ϕ_{max} hence the expressions for the densities in region 1 and region 2 should agree at $\phi = \phi_{max}$.

To the left of the hill: $n1_i(2,2) = 0.677$ To the right of the hill: $n2_i(2,2) = 0.677$

In the program loop, an if statement will be used to select n_1 if $x < x_{max}$ and n_2 otherwise.

The ion current to the collector is the current of ions that have passed over the potential hill. Hence the integral for the current begins at the ion velocity corresponding to ϕ_{max}.

$$J_i(\phi_{max}) := \int_{\sqrt{\frac{\phi_{max}}{\alpha}}}^{\infty} f_i(v,0)\cdot v\, dv$$

$J_i(0) = 0.066$ The current emitted at the surface.

$J_i(2) = 8.91 \times 10^{-3}$ The ion current passing over a potential hill $\phi_{max} = 2$.

Note that the ion current is reduced as ϕ_{max} is increased.

The electron density and electron current

The electron are accelerated in region 1 and decelerated in region 2. In region 1, the expression for the electron density is the same expression used previously, except that electrons with energy greater than $-\phi_{min}$ at the collector are not present in the left-going distribution. Away from the collector, these fastest electrons have energy $(\phi-\phi_{min})$, thus the integral for the density of left-going particles is cut off at this energy.

Thus for $\phi > 0$ and $\phi_{min} < 0$ the density in region 1 is:

$$n1_e(\phi,\phi_{min}) := \int_{\sqrt{\phi}}^{\infty} f_e(v,\phi)\,dv + \int_{-\sqrt{\phi-\phi_{min}}}^{-\sqrt{\phi}} f_e(v,\phi)\,dv$$

The first of the integrals is for the right-going electrons and the second is for the left-going electrons.

To the right of the maximum, **region 2**, the above expression applies as long as the potential ϕ is positive. At locations where the potential is negative, the lower bound on the integral should be replaced by zero. The expression above can be revised to be correct in both reions by replacing ϕ in the lower bound by max(0,ϕ) which prevents negative values for ϕ.

$$n_e(\phi,\phi_{min}) := \int_{\sqrt{\max(0,\phi)}}^{\infty} f_e(v,\phi)\,dv + \int_{-\sqrt{\phi-\phi_{min}}}^{-\sqrt{\max(0,\phi)}} f_e(v,\phi)\,dv$$

We can test this expression by using a ϕ_{min} value that reflects nearly all electrons and several values for ϕ:

$n_e(0,-8) = 1$ The density at the emitter surface if most electrons are reflected.

$n_e(0,-3) = 0.993$ The density is less if fewer electrons are reflected.

$n_e(-1,-3) = 0.36$ The density near the collector where the potential is -1.

$n_e(1,-3) = 0.421$ The density in the quasineutral plasma if the plasma potential is 1.

The above expression for n_e applies in region 1 and region 2.
The electron current to the collector at any negative potential is

$$J_e(\phi) := \int_0^{\infty} f_e(v,\phi)\cdot v\,dv \qquad J_e(0) = 0.282$$

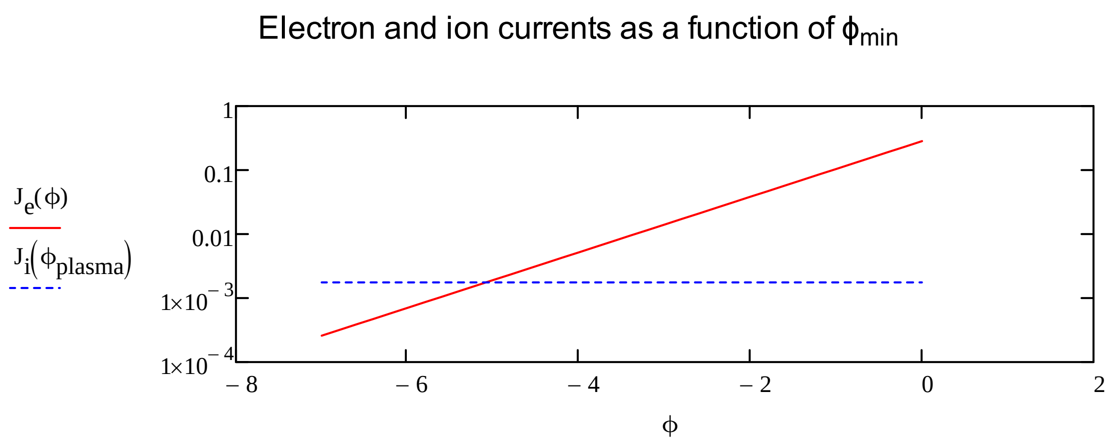
This value agrees with the analytic result for a Maxwellian distribution: $\frac{1}{2}\cdot\sqrt{\frac{1}{\pi}} = 0.282$

The expression simplfies to: $J_e(\phi) := \frac{1}{2}\cdot\frac{1}{\sqrt{\pi}}\cdot e^{\phi} \qquad J_e(0) = 0.282$

The floating potential of the collector:

The floating potential is the potential that causes the collected current to be zero.
We can find this potential by plotting the electron and ion currents as a function of potential:

$\phi := -7, -6.8 .. 0$ ϕ values for plotting.



An accurate value for the collector potential is found using the root finder in the region –8 to 0.

$$\phi_{collector} := \text{root}\left(J_e(\phi) - J_i(\phi_{plasma}), \phi, -8, 0\right) \qquad \phi_{collector} = -5.082$$

Above we have ignored the potential barrier above the hot plate that reduces the ion current. In the program loop, we will use the **max** function to find the most positive potential at each iteration and calculate the ion current taking into cosideration the sheath at the source.

Define the x grid

The sheath solution has more detail than the one obtained previously so we will increase the resolution by changing Δx from 1 to 0.5 and we will increase the number of points.

$xmax := 30$ The size of the domain. $\Delta x := 0.5$ Grid spacing

$kmax := \frac{xmax}{\Delta x}$ $k := 0, 1 .. kmax$ $kmax = 60$ There will be kmax+1 grid points.

$x_k := k \cdot \Delta x$ The grid points.

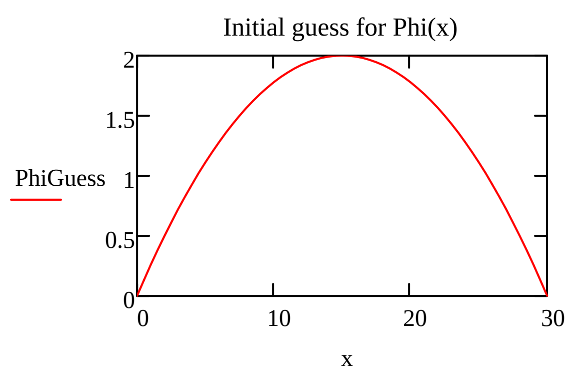
$iters := 640$ The number of iterations in the program loop (a guess).

The trial solution for Φ for the first iteration

Our initial guess for the answer will be a parabola with a maximum at +2.

$$PhiGuess_k := -8 \cdot \left(\frac{0.5 \cdot x_{kmax} - x_k}{x_{kmax}} \right)^2 + 2$$

At right is a plot of the initial guess.



In our program loop we will need to find the location x_{max} of the maximum in the potential. This will be done using the **max** function to first find Φ_{max}.

We will test the procedure and see that it works correctly:

$phimax := max(PhiGuess)$ $phimax = 2$

The lookup function is used to find the value of x corresponding to phimax:

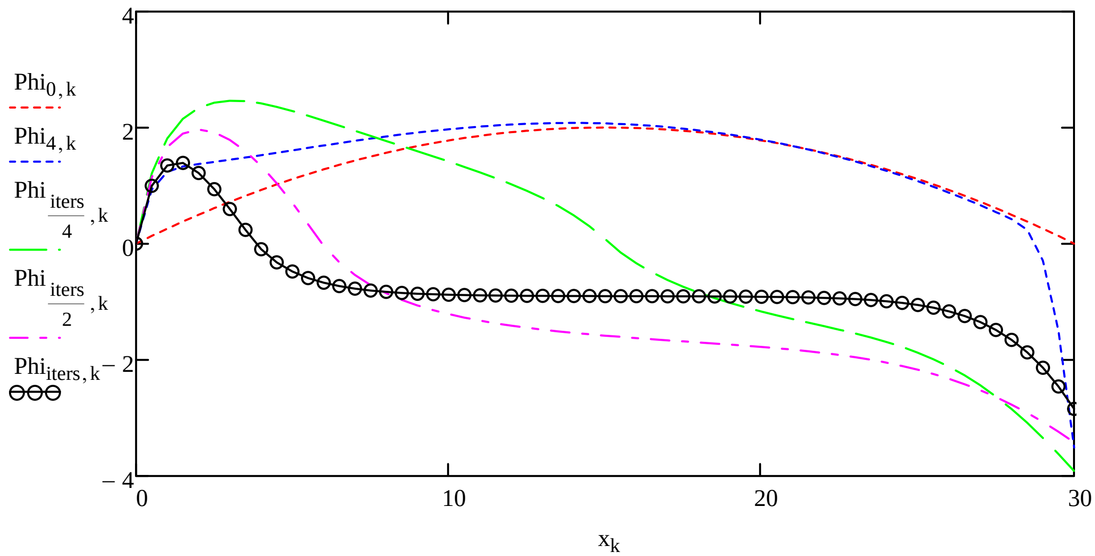
$xmax := lookup(phimax, PhiGuess, x)_0$ $xmax = 15$

The program loop:

$$
\text{Phi} := \left|
\begin{array}{l}
\text{Phi}_{\text{iters},\,\text{kmax}} \leftarrow 0 \\
\text{for } \ k \in 0\,..\,\text{kmax} \\
\quad \text{Phi}_{0,\,k} \leftarrow \text{PhiGuess}_k \\
\text{for } \ i \in 1\,..\,\text{iters} \\
\quad \left|
\begin{array}{l}
\text{phimax} \leftarrow \max\left[\left(\text{Phi}^T\right)^{\langle i-1\rangle}\right] \\
\text{xmax} \leftarrow \text{lookup}\left[\text{phimax},\left(\text{Phi}^T\right)^{\langle i-1\rangle},x\right]_0 \\
\text{phimin} \leftarrow \text{root}\left(J_e(\phi) - J_i(\text{phimax}),\phi,-8,0\right) \\
\text{for } \ k \in 1\,..\,\text{kmax} - 1 \\
\quad \left|
\begin{array}{l}
N_i \leftarrow \text{if}\left(x_k < \text{xmax},\text{n1}_i\left(\text{Phi}_{i-1,\,k},\text{phimax}\right),\text{n2}_i\left(\text{Phi}_{i-1,\,k},\text{phimax}\right)\right) \\
C \leftarrow 0.5\cdot\left(\text{Phi}_{i-1,\,k+1} + \text{Phi}_{i-1,\,k-1}\right) + 0.5\cdot\Delta x^2\cdot\left(-n_e\left(\text{Phi}_{i-1,\,k},\text{phimin}\right) + N_i\right) \\
\text{Phi}_{i,\,k} \leftarrow 0.5\cdot\left(C + \text{Phi}_{i-1,\,k}\right)
\end{array}
\right. \\
\text{Phi}_{i,\,\text{kmax}} \leftarrow \text{phimin} \\
\text{Phi}_{i,\,0} \leftarrow \text{Phi}_{i-1,\,0}
\end{array}
\right. \\
\text{Phi}
\end{array}
\right.
$$

Note that in the program loop periodic boundary conditions are not used; fixed boundary conditions are used. The value assigned to the right boundary, phimin, is the potential for zero current collection. This is continually updated using the root finder because it is dependent upon the the maximum potential, phimax, that is changing with each iteration.

The answer matrix Phi is plotted below for several iterations



The potential profile for the single-ended Q machine is very different from that of the double-ended Q machine. For our ion-rich conditions, there is a potential hill near the emitter (the source sheath). At the right boundary, there is a collector sheath. Below we will look at the electron and ion densities in these sheaths.

Place the final profile into a vector phi for convenience: $phi := \left(Phi^T\right)^{\langle iters \rangle}$

The maximum potential is: $phimax := max(phi)$ $phimax = 1.39$

This occurs at x value: $xmax := lookup(phimax, phi, x)_0$ $xmax = 1.5$

The minimum potential is: $phimin := min(phi)$ $phimin = -2.846$

The potential in the middle of the plasma is: $phi_{\frac{kmax}{2}} = -0.904$

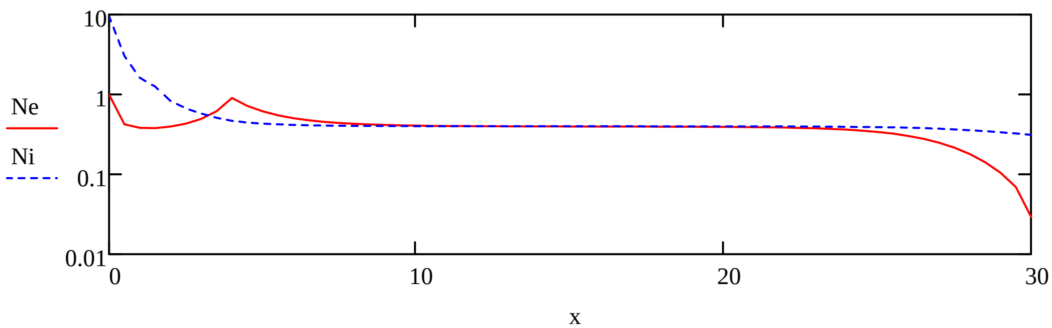
Plot of the electron and ion densities

Below the definitions of the electron and ion densities have been copied from the program loop and given new names:

$Ni_k := \text{if}\left(x_k < xmax, n1_i\left(phi_k, phimax\right), n2_i\left(phi_k, phimax\right)\right)$ The ion density.

$Ne_k := n_e\left(phi_k, phimin\right)$ The electron density.

Log plot of electron and ion densities



The double layer

In the plot of densities, there is an excess of ions adjacent to the emitter and to the right of this there is an excess of electrons. These adjacent layers of opposite charge are a double layer. At the right boundary there is an excess of ions that is an ion sheath.

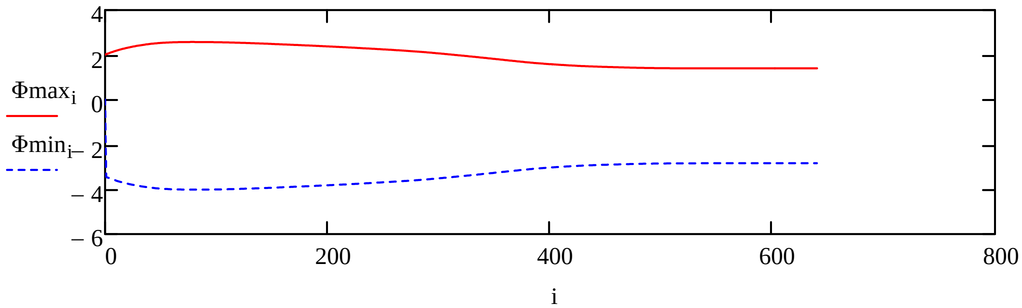
Test for convergence

Did we use a large enough value for imax to converge to the correct answer?
We can answer this question by watching the evolution of the maximum in Φ.

$i := 0 .. iters$ Index for plotting successive value of Φ_{kmax}.

$\Phi max_i := \max\left[\left(Phi^T\right)^{\langle i\rangle}\right]$ $\Phi min_i := \min\left[\left(Phi^T\right)^{\langle i\rangle}\right]$

This is a plot of Φ_{max} and Φ_{min}, as a function of iteration number showing convergence.



Inspection of the density plot shows the following. The ion density is near the maximum value (10) at the source because most of the ions are return by the positive sheath above the source. The ion density is constant throughout most of the central region, then falls (as a consequence of the continuity equation) in the collector region where the ions are accelerated. The electrons at the source are accelerated by the source sheath which reduces their density (again as a consequence of the continuity equation). The density of elecrons goes back to the source value when the potential returns to the source potential, then falls as the potential dips below zero near x = 4.

Notes

1. In this exercise, some of the integrals could have been expressed as error functions.
2. Sheaths may be unstable.
3. If there are collisions, dips in the potential may fill with ions and be destroyed. Simiilarly, potential maximima may fill with electrons.

References

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